Combination of Simulation and Optimization in the Design of Electromechanical Systems

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Summary:

The standard approach in the combined usage of simulation and optimization relies on the application of a black-box coupling of simulation and optimization software. This approach has been applied successfully in many different areas. However, especially in coupled physical systems, the application of this approach may be limited by efficiency considerations.

To overcome these shortcomings of the black box approach, we have recently coupled simulation and optimization on the equation system level. This approach, which has been implemented into the CAPA simulation program, enables the use of semi-analytical gradient calculations and, therewith, may lead to significant time savings by providing highly accurate results.

Keywords:

Coupled systems, non-linear optimization, semi-analytical gradient calculations
1 Introduction

Electromechanical transducers, working as sensors and actuators, play an increasing role in a large variety of applications, ranging from process technology, automotive industry, electro medicine, to consumer products. In order to speed up the design of these devices, the use of simulation tools which are based on numerical methods like the FEM has become more and more attractive. Due to the presence of different electromechanical coupling effects, the simulation of such a device typically exhibits significant complexity in its own, especially when nonlinearities have to be taken into account. Furthermore, the development of an electromechanical transducer is usually also constrained by certain boundary conditions regarding, for example, functionality, shape, or both. Therewith, an optimization problem for a coupled field problem has to be considered in the design process of such an electromechanical device.

The standard approach in solving this task consists in applying a black-box coupling of simulation and optimization software and is applied successfully in many different areas. However, especially in problems involving coupled physical systems, such as electromechanical transducers, the application of this approach may be limited by efficiency considerations. Problems with a solution time of only 1 hour for a single simulation may easily lead to calculation times of several weeks in case of a complex problem with several optimization parameters.

To overcome these shortcomings of the black box approach, we have recently coupled simulation and optimization on the equation system level. This approach, which has been implemented into the CAPA simulation program, enables the use of semi-analytical gradient calculations and, therewith, may lead to significant time savings by providing highly accurate results.

The rest of the paper is organized as follows. First, we will present the necessary theoretical background and compare the setup for the black-box coupling and the semi-analytical approach. To study the applicability of the implemented algorithms, several application examples will be presented, involving optimization of both geometry as well as material parameters. In the case of a CMOS microphone we will consider optimization of the sensitivity, resulting in a multi-objective optimization problem, which can be solved by combined solutions of static and eigenvalue problems. As a second application we investigate the optimization of the sound pressure level of an electro-dynamic loudspeaker. In this case a time-domain simulation must be solved, whereas the objective function is formulated in the frequency domain. Finally, we demonstrate the application of this combined simulation scheme to model reduction problems.

2 Theory

The computer based optimization of an electromechanical transducer requires efficient simulation algorithms coupled to automated optimization solutions. In the approach taken here, we use the simulation capabilities available in CAPA and combine these with sophisticated optimization solution in different ways. The theory of the underlying FEM-BEM algorithms for the simulation scheme is well known and has already been published elsewhere [1,2]. Therefore, only the optimization algorithms and the background required for coupling of simulation and optimization is presented here.

2.1 Optimization

Most real-life design optimization problems require the simultaneous optimization of several possibly conflicting objective functions. These multi-objective optimization problems [3] can be mathematically formulated as follows:

$$\min_{x \in \mathcal{F}} \quad F(x) = (f_1(x), \ldots, f_k(x))^T$$

where $k \geq 2$ and

$$\mathcal{F} = \{ x \in \mathbb{R}^n : g_j(x) \geq 0, j = 1, \ldots, m, \quad x_i \leq x_i \leq x_i, \quad i = 1, \ldots, n \}.$$  

denotes the set of feasible solutions, which is assumed to be non-empty. In contrast to single-objective (scalar) optimization problems, in multi-optimization problems there is usually no solution $x^*$ which is optimal for all objective functions. Thus, relaxed optimality conditions are required.

One approach to overcome this difficulty relies on the notion of pareto optimality. Here, a point $x^*$ is called pareto optimal for (MOP), if no criterion can be improved without worsening at least one other criterion. Usually, this notion of optimality leads to multiple optimal solutions, the so called pareto front. The multi-objective problem is almost always transferred into one or a sequence of single objective or scalar optimization problems (scalarization) whose solutions are pareto optimal for problem (MOP). Those scalar problems are then solved by efficient and reliable algorithms for single objective
nonlinear optimization. In the optimization modules used in our software environment, numerical methods for multi-objective optimization based on \textit{a priori} and \textit{a posteriori} preferences are available. \textit{A posteriori methods} are methods for generating pareto optimal solutions by scalarization after the pareto set (or a part of it) has been generated. One option is the so called \(\epsilon\)-constraint method, where one of the objective functions is selected to be optimized and the other objective functions are converted into constraints by setting an upper bound to each of them. \textit{A priori methods} are methods where the preference must be specified before the solution process. One possibility is the so called \textit{preference or value function} approach, where an accurate and explicit mathematical formulation of a scalar preference or value function which represents the preference globally must be given. The difficulty is that for most problems the engineer does not necessarily know beforehand what is possible to attain in the problem and how realistic her or his expectations are. As indicated above most deterministic mathematical schemes for the solution of multi-objective optimization problems are based on solving one or a sequence of multiple single objective optimization problems. Several different categories of optimization algorithms can be distinguished. Due to its efficiency and robustness (especially for the problems considered in the context here), we will only look at so-called sequential quadratic programming (SQP) methods. These have been established as the standard general purpose tool for solving smooth nonlinear optimization problems under the following assumptions:
- the problem is not too large
- the functions and gradients can be evaluated with sufficiently high precision
- the problem is smooth and well-scaled
- there is no further model structure that can be exploited.

Detailed discussions about SQP methods are contained in [4], and numerical comparative studies show their superiority over other mathematical programming algorithms under the assumptions mentioned above. The fundamental idea behind SQP is to generate a sequence of quadratic programming subproblems obtained by a quadratic approximation of the Lagrangian function and a linearization of the constraints. Second order information is updated by a quasi-Newton formula and the method is stabilized by an additional line search. Among the most attractive features of SQP methods is the superlinear convergence speed in the neighbourhood of a solution.

### 2.2 Coupling optimization and simulation

The typical approach in coupling simulation and optimization is the so-called \textit{black-box approach}, the setup of which is shown in figure 1. As can be seen, the optimizer takes control of the problem. Depending on the actual value of the target function and the gradient with respect to the optimization parameters, new parameter values are defined and a function evaluation is requested. The simulation program is called which just runs the calculation and returns a new function value to the optimizer. The calculation of the gradient of the target function, which is required for a new parameter estimation, is based on calculation of numerical derivatives (finite differences) and completely in the responsibility of the optimizer. Accordingly, several function evaluations are required for each finite difference calculation and, especially in the case of several optimization parameters, a high number of function evaluations may be required for the calculation of the gradient. In a semi-analytical approach, however, the situation is different (cf. figure 2). Here, the simulation not only returns the function value but also the partial derivatives of the function and, therewith, the gradient of the target function with respect to the optimization variables. As a consequence, no gradient calculation is required by the optimizer and the number of calls of the simulation program can be highly reduced.

![Figure 1: Principle setup of the black-box approach](image)
The newly developed simulation scheme relies essentially on a differentiation of the basic system of partial differential equations with respect to the optimization parameters. In the case of coupled systems, this process leads to several new systems of equations, which have to be solved in order to calculate the derivative of the objective function with respect to the optimization parameters. The principle setup of our approach is shown in figs. 3 and 4, where the computational flow for a simulation of a coupled magneto-mechanical system is shown with and without optimization coupling. It is interesting to note, that the new equation systems differ from the original system essentially in the calculation of the right hand side, and, therewith, can be solved with the same system matrices as the original systems. As a consequence, the additional computational costs can be kept pretty low, and an extremely efficient calculation scheme is derived.

As an application, consider the optimization of an electrodynamic loudspeaker with voltage driven coil. In this case, the resulting equation systems, which have to be solved, are given in detail by:

### Original Magnetic system of equations

\[
\begin{align*}
\left( M_d - f_d \right) \ddot{A} + \left( K_d \right) A &= R_d \\
\left( f'_d R_s \right) i &= 0
\end{align*}
\]

### Original mechanical system of equations

\[
M_d \ddot{d} + C_d \dot{d} + K_d d = f_d(A) - M_d \ddot{d} - C_d \dot{d} - K_d d
\]

### Additional magnetic system

\[
\begin{align*}
\left( M_{d,p} - f_{d,p} \right) \ddot{A}_{p} + \left( K_{d,p} \right) \ddot{A}_{p} &= 0 \\
\left( f'_{d,p} R_{s,p} \right) i_p &= 0
\end{align*}
\]

### Additional mechanical system

\[
M_{d,p} \ddot{d}_{p} + C_{d,p} \dot{d}_{p} + K_{d,p} d_{p} = f_{d,p}(A) - M_{d,p} \ddot{d}_{p} - C_{d,p} \dot{d}_{p} - K_{d,p} d_{p}
\]

### 3 Optimization of a CMOS Microphone Membrane

Integrated CMOS microphones offer several advantages when compared with standard microphone setups. Miniaturization, high temperature stability, low power requirements, as well as low cost and mass production are only some issues, which are worth mentioning. The applicability of such devices, on the other hand, strongly depends on the sensitivity and the signal to noise ratio which can be achieved. The sensitivity of the microphone, in turn, depends on the corresponding one of the used...
membrane. In our final study presented here, we have therefore applied our newly developed combination of simulation and optimization software to the optimization of a special membrane design.

![Image](image1.png)

**Fig. 5: Principle setup of a CMOS microphone**

**Fig. 6: Principle setup of finger type membrane**

The principle setup of the types of microphones we consider is shown in fig. 5. A flexible membrane electrode, which usually is made of polysilicon, is mounted along its boundary. A small air-filled gap separates the flexible electrode from the stationary counterelectrode. Typical dimensions of such a device are membrane radius 500-1000 µm and a gap height of 100-200 nm. The flexible membrane is subject to considerable prestressing, and, therewith, the sensitivity of the membrane to pressure loads must be considered seriously. In order to increase the sensitivity, several design modifications besides the simple circular plate have been proposed, like finger or corrugated membranes. Here, we will only consider finger type membranes and the principle setup of such a membrane is shown in fig. 6. Each membrane is defined by its circumference radius \( R \), the inner radius \( R_0 \) of the membrane, the membrane thickness \( t \), the number of fingers \( n \) as well as the width \( b \) of the fingers. Only membranes with 4, 8, and 16 fingers have been considered in the simulation runs.

As the microphones are usually also required to have a large bandwidth, the design goal chosen consisted in the simultaneous optimization of bandwidth and sensitivity of the membrane. Therewith, this problem corresponds to a typical multi-objective optimization problem as discussed above. In order to apply the afore mentioned methods, we had to choose an appropriate scalarization of this multi-objective optimization problem. For this it should be noted that the bandwidth is limited by the first resonance of the membrane, whereas the sensitivity can be described by the static deflection of the membrane due to a specified pressure. Therefore, we considered the product of the first eigenfrequency \( f_1 \) of the membrane and the centre deflection \( u_c \) due to a fixed pressure as our scalar optimization function.

![Image](image2.png)

**Fig. 7: Optima for scalar target function for 4, 8, and 16 finger membranes (R in mm)**

**Fig. 8: Pareto curve, 4 finger membrane, R=500 µm (frequency given in kHz).**

For every membrane the circumference radius \( R \) as well as the thickness \( t \) have been fixed parameters during the optimization runs. The radius \( R \) was chosen to be 400 µm, 500 µm, 600 µm, 800 µm, and 1000 µm and, since we considered 4, 8, and 16 fingers, we ended up with a grand total of 15 scalar optimization problems:

\[
\min_{b, R_0} F(b, R_0) = -f_1(b, R_0) \cdot u_c(b, R_0).
\]

These optimization problems have been solved employing a SQP method and due to its fast local super-linear convergence behaviour those problems have all been solved in only a few iterations. Due
to this convergence behaviour and the short calculation times a black-box approach could be successfully applied in this optimization problem.

In fig. 7 the obtained optimum values for the target function are depicted as a function of the radius $R$ for all numbers of fingers considered. As an example of a pareto front, the $\epsilon$-constraint method described in section 2 above is applied for a 4 finger membrane with $R=500$ $\mu$m. The resulting Pareto curve is shown in fig. 8 and gives the static centre deflection as a function of the first eigenfrequency, or, in other terms, the sensitivity of the membrane as a function of the bandwidth.

![Fig. 9: Principle setup of an electrodynamic loudspeaker](image)

**4 Electrodynamic loudspeaker**

**4.1 Sound pressure level**

As a second application we consider the optimization of the sound pressure level (SPL) of an electrodynamic loudspeaker. The principal setup of such a loudspeaker is shown in figure 9, whereas a corresponding finite element model is depicted in figure 10. The simulation of this coupled field problem requires the simultaneous solution of the coupled magnetic, mechanical, and acoustic field problems as well as their mutual couplings.

![Fig. 10: Finite element model for SPL calculation](image)

In the optimization problem considered here, the primary target was to obtain an SPL as constant as possible in the frequency range from 300 Hz to 10 kHz. As the material data was fixed, the optimization parameters were given by the geometry parameters of the loudspeaker. Since the optimization requirement consisted of a constant SPL, without specifying the actual value, an additional parameter was introduced in the optimization – the value of the unknown SPL. Since the simulations are run in the time domain, the sound pressure is calculated as a time-varying signal. In
order to obtain the frequency dependency of the SPL, a Fourier transform must be added and the resulting level must be referred to the spectrum of the driving electric voltage pulse. The results of the optimization runs are shown in figure 11 and compared to the initial setup. The variation range of the SPL could be reduced from $86 \pm 4$ dB to $85.5 \pm 2.5$ dB in the frequency range from 300 Hz to 10 kHz, and even more from $86 \pm 4$ dB to $84.5 \pm 1.5$ dB when restricting the frequency range to $2 – 10$ kHz.

![Figure 11: Optimization of loudspeaker SPL: Original (blue), intermediate result (green), optimum (red)](image)

These simulation results have also been obtained by application of the black-box approach. Since the acoustic model has to be adapted to the frequency range, the discrete model must be able to handle frequencies significantly higher than the 10 kHz required in the objective function; we adjusted the model to cover the whole audio range up to 20 kHz and thus obtained reliable and accurate results for the objective function, and, especially, the gradients. However, this approach of course leads to large models (200,000 elements typical) and significant simulation times (2 h typical for a single simulation run). Therewith, significant calculation times for the complete optimization problem result.

4.2 Model reduction

This new combined calculation scheme can also be used in model reduction considerations. In case of the loudspeaker we have considered linearization of the nonlinear magnet assembly and replacement of the ambient fluid. With these 2 modifications, tremendous speed-up can be obtained in the calculation of the distortion factors.

For the linearization problem, first a complete nonlinear simulation is run, giving the distribution of the magnetic field inside the gap of the assembly. Next, the magnet assembly is partitioned into 5 different regions (cf. figure 12), and the optimization parameters are given by the magnetic permeability of the regions. Using the nonlinear solution of the magnetic field in a given set of nodal points $n_{sol}$ as the target value, a simple least square optimization problem can be defined by

$$f(p) = \frac{1}{n_{sol}} \sum_{i} (u_i(p) - u_{nonlin}^2)$$

It must be noticed, that the solution of the optimization problem significantly depends on the set of nodal points, which is considered in the objective function. This set must not be too small and should also contain several points not inside the air gap. This optimization problem has been treated using the black-box as well as the semi-analytical approach. While both solutions gave nearly identical results, a speed-up factor 15 was observed using the semi-analytical approach. The comparison of the result of the nonlinear solution and the optimum of the semi-analytical optimization is shown in figure 13.
A similar approach can be used, to replace the ambient fluid by a set of spring elements which are located on the fluid-solid interface of the original model (cf. figure 14). Here, we used as objective function the vibrations of the coil which had been calculated in a reference run including the fluid domain. The final results of these optimizations can be seen in figure 15, where again good agreement is found. Here a somewhat smaller speed-up factor between 6 and 10 when comparing semi-analytical and black-box approaches has been observed.

5 Conclusion

We have successfully applied a new calculation scheme to the optimization of electromechanical transducers. This scheme relies on a tight coupling of simulation and optimization and involves the solution of additional equation systems during the simulation of the transducer. Therewith, not only the value of the objective function is calculated, but also the gradient of the objective function with respect to the optimization parameters is returned. Using this new approach, significant speed-up can be obtained when compared to a standard black-box coupling of optimization and simulation. First results of the application of this new calculation scheme in the design of MRI scanners are reported in [5].

6 References