Numerical Modelling and Design Optimisation of Clinical MRI Scanners

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Summary

In this talk the application of a CAE environment in the design process of clinical magnetic resonance imaging (MRI) scanners is presented. Using the proposed numerical methods improved efficiency in the development of these MRI scanners can be achieved. The simulation scheme is based on a finite element method and allows for the full 3D calculations of the electromagnetic, mechanical, and acoustic field effects including all mutual couplings. Efficient algorithms based on iterative solvers utilizing advanced preconditioners and algebraic multigrid methods are used to compute the complex models. Furthermore, a recent addition allows the modelling of the electric field sources based on an application of the law of Biot-Savart. Therewith, a full FE-model of the complex coil structure can be avoided. This simplification not only reduces CPU time but also modelling time and complexity enormously.

This numerical simulation scheme has recently been combined with automated optimization algorithms. These use a powerful implementation of a deterministic, numerical optimisation algorithm, which is based on a robust SQP-method in combination with an efficient methodology to calculate the gradients of the objective and constraint functions. With this set-up a computer based optimisation of the design of MRI scanners with respect to, e.g. eddy currents losses induced by magneto-mechanical couplings, is made available.

Keywords

Magnetic resonance imaging, magnetomechanical-acoustic system, finite elements, optimization, iterative solvers, algebraic multigrid
0. Introduction

To minimize time and efforts in the development of clinical magnetic resonance imaging (MRI) scanners, precise and efficient computing and simulation tools are necessary. With such computer simulations the expensive and time-consuming fabrication of a large number of prototypes can be reduced. However, as shortly sketched in the following, MRI is a prime example of an application that combines various physical branches ranging from superconducting magnets, the interaction of quantum-mechanical spins with electro-magnetic fields, the generation of highly linear magnetic field gradients to the production of (unwanted) acoustic noise due to the vibration of the MRI scanner.

Two basic components of the MRI scanner which are of interest in the following are the main magnet and the gradient coil. The main magnet generates a strong static field to align the spins of the nuclei in the human tissue [1]. For high quality MR imaging this magnetic field has to be highly homogeneous (typically better than 5ppm) in the region of interest (field-of-view „FoV“). In clinical state-of-the art MRI scanners the magnetic field strength is larger than 1 T, and the main magnet is a superconducting magnet. The gradient coils produce highly linear magnetic field gradients within the field-of-view. These field gradients are needed for the spatial localization of the magnetic resonance signal, and for the selection of the slices. To generate a field gradient along the three orthogonal axes, there exist three sets of gradient coils (X- Y- and Z-coils) exist. Since the rotational axis of the cylinder (which is the basis geometry of the gradient-coil) is parallel to the z-axis, the z-coil essentially consists of rotationally symmetric coils. For the transverse gradient (x- and y-) coils, four three-dimensional saddle coil structures are employed, as displayed in Fig. 1.

As displayed in Fig. 2, where the superconducting magnet and the gradient coils are displayed, the gradient coil is mounted close to the cryostat. The cryostat consists essentially of the stainless steel outer vacuum chamber, aluminium radiation shields, and the stainless steel helium vessel. The helium vessel contains the superconducting coils of the main magnet which have to be hold at the temperature of liquid helium. In a MRI measurement, the gradient coil is driven by an appropriate pulse sequence, leading to a rapid change of the current in the wires of the gradient coils. Since the gradient coil is exposed to the strong static magnetic field of the main magnet, the pulse sequences result in strong Lorentz forces acting on the (gradient coil) wires, and, hence, to vibrations of the gradient coil. Additionally, even with a well-shielded gradient coil, a small residual magnetic stray field in the magnet is generated. Furthermore, the interaction between the magnetic stray field and the strong magnetic field of the magnet results in a very complex response function due to the load by the gradient coil. The physics of this response is governed by strongly coupled Lorentz forces, mechanical vibrations, and eddy currents. For this reason, for the magnetic resonance industry, the following magneto-mechanical-acoustic effects are of great interest in magnetic resonance industry:
• gradient-induced eddy current losses in the cryostat
• noise induced by the vibrations of the gradient coil and the cryostat
• low frequency vibrations of the magnetic field within the bore due to external vibrations
  
  Furthermore, the main interest in the design of gradient coils is a fast switching time (i.e., a low inductance), a high linearity in the field of view, and vanishing effects in the field-of-view due to magnetic stray fields.

Due to the intricate physics, any computer simulation tool for clinical MRI scanners has to account for at least some of the multiphysical effects, and, in particular, for the coupling of different fields.

Standard computer modelling tools based on finite and/or boundary elements are nowadays well suited in the design of pure electromagnetic or pur mechanical-acoustic field problems. Furthermore, the main interest in the design of gradient coils is a fast switching time (i.e., a low inductance), a high linearity in the field of view, and vanishing effects in the field-of-view due to stray fields.

However, commercially available general purpose computer simulation packages cannot be efficiently employed to solve complex interactions of various fields, as necessary as needed in an industrial development and design process, to solve the complex interactions of the above described fields as it is necessary to tackle the above sketched effects.

Here, a CAE environment for a more efficient development of clinical MRI scanners is presented. The modelling scheme is based on a finite element method and allows the full three-dimensional calculation of the magneto-mechanical-acoustic behaviour of a MRI scanner, including the mutual couplings of the different fields. The scheme has been implemented in the finite element/boundary element program CAPA [5]. In addition, SIEMENS in-house routines are employed to calculate the magnetic-field effects of the gradient coils of the MRI scanner.

The initial implementation of the simulation environment [2] requires a full modelling of the three-dimensional wires of the gradient coil by means of finite elements. To obtain converged results, this means that a high number of finely discretized finite elements is needed, a task that results in both, long calculation times, and a high and time-consuming modelling effort. To overcome this problem, the sources of the electric fields are now modeled based on the laws of Biot-Savart. This ansatz allows a much faster modelling, and, particularly a much faster change of the model. Thus, it is now possible, to combine the simulation environment with automatic optimization algorithms, with the (electric) model of the gradient coil as the design variables.

In this paper, at first the computation scheme is presented with emphasis, especially on the new ansatz based on the laws of Biot-Savart is presented. Next, we demonstrate the applicability of the simulation environment is applied to an industrial design example. Finally, we show how the combination of this CAE environment can be combined with automatic, deterministic optimization algorithms and — This combination concludes presented with an example of the optimization of a MRI scanner with respect to eddy current losses.

1. Computation scheme

1.1 Governing equations

According to the discussion above, in the computer simulation of a clinical MRI scanner the following physical fields (including their mutual couplings) have to be modelled:

- Magnetic field
- Mechanical field in a solid
- Acoustic field in a fluid
- Magneto-mechanical coupling
- Fluid-solid interactions

In this paper the governing equations are solved using finite elements methods. To reduce CPU-time efficient solvers based on enhanced preconditioners and algebraic multigrid methods (AMG) are employed for the magnetic field problem. For the mechanical (resp. the mechanical/acoustic system) a GMRES algorithm including advanced preconditioning techniques is applied. This combination of solvers has proven to supply a stable solution of the fully coupled transient simulation. A detailed discussion of the underlying theory, can be found in [2]
1.2 Utilizing Biot-Savart's Law

A standard finite-element modelling of MRI scanners necessitates a full three-dimensional model of the wires of the gradient coil. However, the physics inside the superconducting magnet only depends on the sources of the gradient field. Hence, the simulation environment was modified such, that the conductors of the gradient coil are replaced by an equivalent set of line currents. The resulting magnetic field intensity $\mathbf{H}_{GC}$ can be computed in advance by applying Biot-Savart's law (see, e.g., [4])

$$\mathbf{H}_{GC}(x', y', z') = \frac{1}{4\pi\mu_0} \int I_{GC} \frac{r}{r^3} \times \mathbf{J}' \, dI$$

(1)

with the field (observation) point $(x', y', z')$, $I_{GC}$ the equivalent line currents of the gradient coils, and $r$ the distance to the observation point. Therewith, we decompose the magnetic field $\mathbf{H}$ into the magnetic field $\mathbf{H}_{GC}$ of the gradient coils and the magnetic field $\mathbf{H}'$ of the superconducting magnet coils as well as the reaction field

$$\mathbf{H}' = \mathbf{H}_{GC} + \mathbf{H}'.$$  

(2)

In addition, we have to decompose the magnetic vector potential according to

$$\mathbf{A} = \mathbf{A}_{GC} + \mathbf{A}' = \mathbf{H}_{GC} + \mathbf{H}'.$$  

(3)

Therewith, the formulation for the magnetic vector potential $\mathbf{A}'$ reads as follows

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A}' \right) = \mathbf{J}' - \nabla \times \mathbf{H}_{GC} - \rho \mathbf{A}.$$  

(4)

With this decomposition, we can in a first step compute the magnetic field $\mathbf{H}_{GC}$ generated by the gradient coils using (1), and in a second step solving (4). Applying this decomposition results in the following expression for the magnetic induction $\mathbf{B}$:

$$\mathbf{B} = \nabla \times \mathbf{A}' + \mu_0 \nabla \times \mathbf{H}_{GC}.$$  

(5)
2. Verification of the modelling scheme

The initial verification of the 3D magnetomechanical computation scheme with a gradient coil discretized by finite elements has already been reported in [2]. Here, in the original model the current-loaded conductors of the gradient coil and the magnet coil are discretized using pure magnetic coil elements. The cryostat as well as a small ambient region are modeled using magnetomechanical elements based on the moving-mesh method [3]. Finally, the outer surrounding air is modeled by pure magnetic elements. Due to the symmetry of the gradient coil and the superconducting magnet, this computer model was simplified by applying proper boundary conditions at the border of a fourth model.

![Fig. 4 3D magneto-mechanical finite element model of a MRI scanner with transverse gradient coil.](image)

For the design of the cryostat, the frequency dependence of eddy current losses is an important parameter. For the computation of this frequency response, a dynamic analysis using a chirp excitation signal with the desired starting frequency as well as a certain sweep rate for the current of the gradient coil was performed. The number of time steps in the simulations (1750) is appropriate to this chirp function. Further input parameters are the geometry of the MRI scanner, the density, modulus of elasticity, Poisson's ratio, and loss-factor for the mechanical materials as well as the electrical conductivity for the magnetic parts. After the computation of the response signals (average eddy current losses in the shields of the cryostat), the time axis was transformed in the corresponding frequency axis. In these computer simulations an AMG solver has been used to solve the magnetic field equations, whereas a GMRES algorithm including an advanced ILU(4) pre-conditioner (fill-in level 4) has been applied to the mechanical system [2].

Due to possible convergence problems, the conductors of the transverse gradient coil and, therefore, the surrounding air has to be discretized very fine. Consequently, a large number of magnetic finite elements as well as a high modeling effort is necessary in the simulation of the MRI-scanner (see Fig. 4). Therefore, a modified finite element model has been applied recently, in which the 3D magnetic coil elements for the gradient coil were eliminated completely. The electromagnetic excitation of the gradient coil is now realized by an equivalent set of line currents (see Section 1.2). With this amendment, the complexity of the computer model as well as the required computer resources are reduced **tremendously** as shown in Tab. 1.

**Table 1 Comparison of computer resources of full model and coarse model utilizing Biot-Savart's law.**

<table>
<thead>
<tr>
<th>model</th>
<th>finite elements</th>
<th>unknowns</th>
<th>physical memory</th>
<th>CPU time per time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>full FE-model</td>
<td>290000</td>
<td>450000</td>
<td>4900 Mbyte</td>
<td>5.6 min.</td>
</tr>
<tr>
<td>coarse FE-model</td>
<td>164000</td>
<td>290000</td>
<td>4100 MByte</td>
<td>3.6 min.</td>
</tr>
</tbody>
</table>
The coarse finite element model utilizing Biot-savart's Law described above has been verified by comparing simulated eddy current losses inside the helium vessel with the corresponding results of the full FE-Model as well as with measured data (see Fig. 5). It should be noted that the complexity of the magnetomechanical problem does not allow any analytic calculations, and therefore, analytical results cannot be used for verification purposes. Considering that the deviations of subsequent measurements of the eddy current losses were within a range of \(\pm 15\%\), an acceptable agreement between simulation results and measured data was achieved. Therefore, it can be concluded that the presented finite element scheme is well suited to the computation of the full 3D magnetomechanical behavior of a real MRI scanner.

![Graph comparing measured and calculated eddy current losses](image)

**Fig. 5** Comparison of measured and calculated eddy current losses with the full FE model, and the model utilizing Biot-Savart's law.

3. **Optimization**

As shown above, the results of the simulation agree rather good with measurements. Hence, the simulation scheme – which does not use adjustable parameters – can be used to optimize the MRI scanner.

3.1 **Handcrafted Sensitivity Studies**

In a first step the optimisation was done “manually”, i.e. based on various simulations, sensitivity studies of different designs were performed. In such an iterative manner, very good improvements were obtained regarding eddy current losses and noise of the MRI scanner. The results of this “handcrafted optimization” are shown in Fig. 6.

![Graphs showing optimization results](image)

**Fig. 6:** Optimization of the MRI scanner: a) with respect to eddy current losses, b) with respect to noise inside the patient bore. The shown results are obtained via “handcrafted optimization”.

3.2 **Optimization utilizing Automated Optimization Algorithm**
To facilitate the use of the simulation environment as a design tool in the development process, the simulation scheme was combined with deterministic optimization algorithms for an automated optimization. Since the objective function (e.g. eddy current losses) is neither linear nor quadratic in the design variables (i.e. the electric design of the gradient coil), a nonlinear optimization algorithm has to be employed.

A general nonlinear constraint optimization problem with optimization parameters \( z \) can be written as

\[
\min_{z} \Phi(z), \quad \text{s.t.} \quad g_{i}(z) \leq 0 \quad (i = 1, \ldots, m)
\]

Here, the objective function \( \Phi(z) \) describes the eddy current losses. Hence, \( \Phi(z) \) is written as:

\[
\Phi(z) = \int_{f_1}^{f_2} \omega(f) \left( \int_{\Omega} \rho^{2} d\Omega - Q_{\text{avg}} \right) df,
\]

where \( Q_{\text{avg}} \) denotes the upper level of eddy current losses which is tolerated in the frequency range \([f_1, f_2] \). To improve the flexibility of the formulation, a frequency-dependent weighting function \( \omega(f) \) is introduced. The linear and nonlinear constraints \( g(z) \) in (6) denote the typical design specifications of gradient coils as, e.g., inductance, linearity in a given field-of-view, shielding, power dissipation, etc. These are evaluated by SIEMENS in-house tools, while the objective function is evaluated with CAPA and the according post-processing steps. For the solution of (6) a robust implementation of the SQP-method (sequential quadratic programming) \([6]\) has been employed.

In the first implementation of the optimization scheme, the derivatives of the objective function with respect to the optimization parameters were calculated using higher order finite differences schemes with CAPA as a “black-box-solver”. Such traditional optimization approaches demand at least two CAPA simulation runs for a single derivative (if the simplest finite differences scheme is applied). Later on, the CAE tool was amended in such way that the derivatives of the objective function are calculated by means of semi-analytical methodologies, as it is sketched in Fig.7. Compared to a traditional approach the semi-analytical calculation of all derivatives during the CAPA run leads to a problem-dependent speed up between 5 to 10.

Using the described optimization approach, already after 5 iterations a significant improvement could be achieved. The results of the automated optimization of the MRI-scanner are compared to the manually obtained optimization results in Fig. 7. The slightly different result of both optimization approaches is due to the different weighting of the objective function. Furthermore the optimization history gives a clear indication which design criterion (i.e. which constraint) forces the solution in a specific direction, or which criteria are even contradictory.

FIG. 7: The original simulation scheme of Fig.3 was amended such that also the derivatives of the objective function with respect to the optimization parameters are calculated during each time-step.
4. Conclusions

A numerical scheme based on a finite element method has been developed for computer modeling of the coupled magnetomechanical-acoustic behavior of clinical magnetic resonance imaging (MRI) scanners.

The initial implementation allowed only the 3D magnetomechanical computation of a simplified MRI scanner with z-gradient coil. Meanwhile, the modeling scheme has been extended and now allows the full 3D calculation of the electromagnetic, mechanical, and acoustic fields including their mutual couplings of a real MRI scanner. In order to reduce the CPU-time of the 3D simulations, we have applied efficient solvers based on enhanced pre-conditioners and algebraic multigrid (AMG) methods.

In the 3D calculations an AMG solver is used to solve the magnetic field equations, whereas a GMRES algorithm including advanced preconditioning techniques is applied to the mechanical/acoustic system. This combination of solvers has proven to supply a stable solution of the fully coupled transient simulation. Furthermore, for further reduction of CPU-time and modeling effort, the finite elements for the conductors of the gradient coil are replaced by an equivalent set of line currents. Therewith, those fully coupled fields can now be handled on a standard computer in reasonable time, applied to industrial problems of practical relevance.

The good agreement of measured and simulated results shows impressively the validity of the presented method. Due to the high predictive power of the simulation scheme, the simulation scheme is ideally suited for a numerical optimisation of an MRI scanner. Thus, the numerical simulation scheme is used for an optimisation of the MRI scanner with respect to eddy current losses. This optimisation, originally performed based on “handcrafted” sensitivity studies, can now be run by recently implemented automated optimisation schemes. These use a powerful implementation of a deterministic, numerical optimisation algorithm, which is based on a robust SQP-method in combination with an efficient methodology to calculate the gradients of the objective and constraint functions. Furthermore, the practical applicability of the developed simulation scheme in an industrial computer-aided design process has been demonstrated by the numerical optimization of the MRI system with respect to eddy current losses and the emitted noise.

Fig. 7: Comparison of manually and automatically optimized eddy-current losses

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original state
automated optimisation
manual optimisation

Eddy current losses (W)
Frequency (Hz)

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Consequently, the software tool presented here provides a basis for more efficient development of clinical MRI scanners, since both development time, and costs can be reduced tremendously. Already now, this provides a significant temporal advantage compared to conventional prototyping. An extension of the existing 3D computer model taking into account efficient parallelization techniques for performing the FE-simulations will be established in the near future.

5. References


